Acoustic effects in the nonlinear oscillations of planar detonations

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(Received 18 October 1995)

The role played by acoustic waves in one-dimensional oscillatory instabilities is investigated at moderate overdrives. The mechanism responsible for the instability is similar to the quasi-isobaric one at high overdrives. The overall effect of pressure fluctuations is stabilizing and decreases the oscillatory frequency at the instability threshold. A nonlinear integral equation for the evolution of the detonation velocity is obtained as an asymptotic solution of the reactive Euler equations.

PACS number(s): 47.40.−x, 47.70.Fw, 47.20.−k

I. INTRODUCTION

Due to a large activation energy in the Arrhenius factor of the exothermal reaction rate, the inner structure of a detonation wave is, according to Zeldovich-Neumann-Döring, constituted by a strong inert shock followed by a subsonic reacting flow. Most of the gaseous detonation fronts are cellular, with a pattern changing continuously with time [1]. Attempts to unravel the underlying physical mechanisms have met with a modicum of success. A basic feature is the coupling of transverse pressure waves, downstream of the leading shock, to mechanisms responsible for longitudinal oscillations [2]. One-dimensional oscillatory instabilities, called galloping detonations, have been observed in experiments [3], and reproduced very early by direct numerical simulations of unsteady detonations in plane geometry [4]. The corresponding Hopf bifurcation was also studied by numerical analysis of the linearized equations [5].

A full understanding of these one-dimensional oscillatory instabilities is a prerequisite to the analysis of cellular detonation fronts. In previous work [6], the physical mechanism triggering the galloping detonations has been identified in the limiting propagation regime of a large degree of overdrive. Such a regime of piston-supported detonations (see Fig. 1) corresponds to a large piston velocity yielding a very large Mach number of the shock and a very small local Mach number of the burned gas flow relative to the shock. Consequently a quasi-isobaric approximation is valid downstream of the shock, throughout the entire reaction zone. Then a global mass conservation linking the mass flux across the leading shock to the variation of mass of the gas downstream the shock, yields strong one-dimensional and nonlinear oscillations of the detonation structure [6]. Until now, pressure waves were commonly considered as essential mechanisms of the instabilities of detonation fronts. Our recent analysis [6] shows that this is not the case for the one-dimensional instability at large overdrives, but the question is left open for other propagation regimes. In this paper the analysis is extended to ordinary regimes at moderate overdrives for which the quasi-isobaric approximation is no longer fully satisfactory. Due to a local Mach number of the flow which is not sufficiently small everywhere, pressure waves with non-negligible amplitudes are associated with any unsteady behavior. Their role is studied in this paper by using a perturbative analysis around the quasi-isobaric approximation.

Physical insights into the problem of galloping detonations may be summarized as follows. Coupling of acoustic waves to heat release may lead to instabilities of a reacting flow in a cavity. Owing to gas expansion, local variations of the heat release rate act as distributed volume sources exciting acoustic waves which, in turn, modify the reaction rate. When a thermoacoustic instability develops, the fluctuations of the heat release rate and pressure are positively correlated (Rayleigh criterion), yielding an amplification of standing acoustic waves in the cavity where the combustion proceeds [7]. The oscillatory instability of detonations is different in many aspects. It consists of a periodic variation of both the shock velocity and the distribution of heat release rate [see Fig. 1(b)] with a period of oscillation which is markedly larger than the acoustic time. The feedback loop of the local gas expansion upon the local rate of heat release proceeds through the shock and involves both the pressure and the entropy waves. Due to the temperature sensitivity of the in-

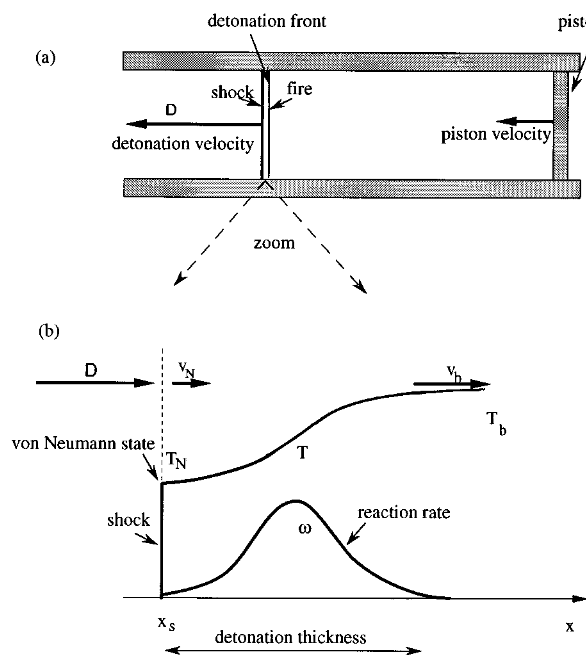

FIG. 1. Plane detonation. (a) Piston-supported detonation propagating in a tube. (b) Sketch of the inner structure of the detonation front. The flow velocities are in the reference frame of the shock.

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duction kinetics, the distance between the shock and the peak rate of the heat release is governed by the temperature variations of the shocked gas associated with the velocity fluctuations of the inert shock. The resulting modifications of the local heat release rate are transported downstream mainly by the entropy waves, while the perturbations carried by the acoustic waves propagating downstream have a smaller influence. However, the pressure waves are responsible for carrying the gas expansion effects upstream, back to the leading shock, for closing the feedback loop. In other words, acoustic waves are essential elements but only as a carrier of information. Furthermore, as shown in this paper, the coupling of the pressure fluctuations to the heat release rate has an overall stabilizing effect upon the oscillatory mechanism. Thus the physical mechanism responsible for the instability is not related to the Rayleigh criterion of thermoacoustic instabilities, but is similar to the quasi-isobaric one exhibited in the limiting case of a large degree of overdrive [6].

In Sec. II we present the formulation of the problem with the basic approximations which are used. The different regimes are described in Sec. III. Section IV contains a perturbative analysis of the pressure effects at moderate overdrives. Finally, the results are discussed in Sec. V.

II. PROBLEM DEFINITION

The objective is to determine the nonlinear evolution of the shock velocity and the detonation structure.

A. Constitutive equations and boundary conditions

According to Rankine-Hugoniot conditions, temperature and pressure perturbations of the shocked gas (Neumann spike), δT_N and δp_N, are associated with small variations of the Mach number M based on the velocity of the shock, δM,

$$\frac{\delta T_N}{T_{N0}} \approx \frac{2(\gamma-1)M_0^2}{(\gamma-1)M_0^2+2}\,\frac{\delta M}{M_0}, \quad \frac{\delta p_N}{p_{N0}} \approx 2\,\frac{\delta M}{M_0}, \tag{1a}$$

valid for a polytropic gas where γ is the ratio of specific heats. The subscripts N and 0 denote, respectively, the state at the Neumann spike and the steady state solution. The strong shock approximation

$$M_0^2 \gg 1, \quad (\gamma-1)M_0^2 = O(1), \tag{1b}$$

which is used in (1a), is valid without restriction in gaseous detonations. The equations of the entropy waves are

$$\frac{1}{T}\frac{DT}{Dt} - \frac{(\gamma-1)}{\gamma}\,\frac{1}{p}\,\frac{Dp}{Dt} = \frac{Q}{C_pT}\,W(Y,T), \quad \frac{DY}{Dt} = W(Y,T), \tag{2}$$

where D/Dt denotes the substantive derivative, and Q is the heat of reaction. In the simplest kinetics model the reaction rate $W(Y,T)$ depends on the gas temperature T and on the progress variable Y only ($Y=0$ at the shock, $Y=1$ in the burned gases). This drastic simplification of the combustion chemistry is used here as an illustrative example; more realistic schemes may be introduced in the analysis. By using a system of reduced coordinates constituded by a mass weighted distance from the shock ξ and a reduced time $\tau = t/t_0$, the substantive derivative takes a simple form in which the convection term involves the instantaneous Mach number of the leading shock $M(\tau)$ only,

$$\xi = \frac{1}{t_0 V_{N0}\rho_{N0}} \int_{x_s(t)}^{x} \rho(x',t)\,dx', \quad \frac{D}{D\tau} = \frac{\partial}{\partial \tau} + \frac{M(\tau)}{M_0}\,\frac{\partial}{\partial \xi}, \tag{3}$$

where the reference time t_0 is the induction time at the Neumann spike of the steady solution, $t_0 \approx W_{N0}^{-1}$ with $W_{N0} \equiv W(Y=0, T=T_{N0})$. The instantaneous shock position is denoted by $x_s(t)$, and v_{N0} is the flow velocity relative to the shock at the Neumann spike of the steady solution, ρ_{N0} is the corresponding gas density. The region downstream of the shock corresponds to $\xi > 0$. The characteristic equations describing the pressure waves are

$$\frac{1}{\gamma p}\,\frac{D^{\pm}}{D\tau}\,p \pm \frac{1}{a}\,\frac{D^{\pm}}{D\tau}\,v = \frac{Q}{C_pT}\,\frac{W}{W_{N0}}, \quad \frac{D^{\pm}}{D\tau} \equiv \frac{D}{D\tau} \pm \alpha\,\frac{\partial}{\partial \xi}, \tag{4}$$

where a is the local sound speed, v is the flow velocity relative to the shock of the steady solution, and $\alpha \equiv (\rho a/\rho_{N0} v_{N0})$ is the reduced sound speed in coordinates (3). The detonation dynamics represented by $M(\tau)$ is obtained by solving Eqs. (2) and (4) with boundary conditions (1) at the shock ($\xi=0$), including $Y=0$ and a Rankine-Hugoniot condition (5) for v, and an additional boundary condition (6) downstream $\xi \to +\infty$. At the same approximation as (1), the Rankine-Hugoniot condition for v yields

$$\xi = 0: \quad \frac{\delta v_N}{v_{N0}} \approx -\frac{1}{(\gamma-1)}\,\frac{\delta T_N}{T_{N0}}. \tag{5}$$

When the size of the burned gas region between the end of the reaction and the piston is much larger than the detonation thickness, a radiation condition is valid for acoustic perturbations transmitted downstream:

$$\xi \to +\infty: \quad \delta p - \rho_{b0} a_{b0} \delta v = 0, \tag{6}$$

where δp and δv are perturbations from the steady state of the burned gas denoted by subscript $b0$. Condition (6) is valid whenever the nonlinear effects of the perturbations in the burned gases are negligible.

B. Basic approximations

When the difference of heat capacities is small, $(\gamma-1) \ll 1$, and when the temperature sensitivity of the induction kinetics is large enough, $\beta_N \gg 1$, the distribution of the heat release rate is modified mainly by the thermal energy transported from the shock by the entropy waves. In the framework of the simple kinetics model (2), one has, by definition,

$$\delta W_N^{-1}/W_N^{-1} \equiv \beta_N \delta T_N / T_N \quad \text{where} \quad W_N \equiv (Y=0, T=T_N). \tag{7}$$

The main effects are picked up in the limit

$$(\gamma-1) \to 0 \quad \text{and} \quad \beta_N \to +\infty \quad \text{with} \quad \beta_N(\gamma-1) = O(1) \tag{8a}$$

and when the attention is limited to weak perturbations of the strong leading shock,

$$\delta M(\tau)/M_0 = O(\beta_N^{-1}) \quad \text{with} \quad (\gamma-1)M_0^2 = O(1). \quad (8b)$$

In such conditions, the pressure term is negligible in (2): heating by reversible adiabatic compressions is much smaller than by the exothermal reaction. Then system (2) forms a closed set of nonlinear equations for the species concentration and temperature which are no longer coupled to the gas dynamics excepted through the boundary condition at the shock. According to (8b), a further simplification occurs when the β_N^{-1} terms are neglected so that the substantive derivative, expressed in coordinates (3), reduces to $D/D\tau \approx \partial/\partial\tau + \partial/\partial\xi$. Within these approximations, the instantaneous distribution of the rate of heat release is obtained in terms of $T_N(\tau)$ by solving

$$\frac{1}{T_{N0}}\left(\frac{\partial}{\partial\tau}+\frac{\partial}{\partial\xi}\right)T = \frac{Q}{C_p T_{N0}} W(Y,T),$$

$$\left(\frac{\partial}{\partial\tau}+\frac{\partial}{\partial\xi}\right)Y = W(Y,T), \quad (9a)$$

valid when β_N^{-1} terms are neglected. For real mixtures the result may easily be obtained numerically as the solution of a system generalizing (9a). In the framework of simplified model (9a), $T = T_N(\tau-\xi) + YQ/C_p$. The distributions of the reduced reaction rate and temperature, $W/W_{N0} = w(\xi,\tau)$ and $T/T_{N0} = \theta(\xi,\tau)$, are obtained at any instant of time, as a nonlinear function of $\delta T_N(\tau)$ only, but at early times involving delays which increase with the distance from the shock. The model used in [6] yields the following analytical expression:

$$w(\xi,\tau) = e^{\Theta_N(\tau-\xi)} w_0(\xi e^{\Theta_N(\tau-\xi)}),$$

$$\theta(\xi,\tau) = \theta_0(\xi e^{\Theta_N(\tau-\xi)}), \quad (9b)$$

with

$$\frac{d\theta_0(\xi)}{d\xi} = \frac{Q}{C_p T_{N0}} w_0(\xi) \quad \text{and} \quad \int_0^\infty w_0(\xi)d\xi = 1, \quad (9c)$$

where $\Theta_N(\tau) \equiv \beta_N \delta T_N(\tau)/T_{N0}$ is the reduced fluctuation of the temperature of the shocked gas, and where $w_0(\xi)$ and $\theta_0(\xi)$ are the reduced distributions of the steady solution. According to (1) and (8), $\Theta_N(\tau)$ is a quantity of order unity. For an ordinary Arrhenius law $W(Y,T) = (1-Y)B\exp(-E_N/RT)$, the brute forced limit $\beta_N \to +\infty$ of the solutions of (2) yields (9b) and (9c), but with a nonsingular steady distribution $w_0(\xi)$ only in the limit of a weak heat release $Q/C_pT_{N0} = O(1/\beta_N)$, while with $Q/C_pT_{N0} = O(1)$ one gets a singular distribution $w_0(\xi) = \delta(\xi-1)$ corresponding to the so-called square-wave model in which the heat release rate is concentrated in an infinitely thin sheet located at $\xi = 1$. The square-wave model is not suitable for studying the detonation dynamics, because it leads to spurious singularities in the high frequency range. As explained in [6], a nonzero thickness of the heat release region is necessary for correctly describing the high frequency behavior. For simplicity, we will use here, as in our previous work [6], a chemical kinetic model defined by (9b) with a smooth distribution $w_0(\xi)$, without restriction on the heat release parameter Q/C_pT_{N0}. The distribution of reduced rate of heat release $w_0(\xi)$ in the steady state is considered independent from the temperature sensitivity of the induction kinetics at the Neumann spike, β_N, and from Q/C_pT_{N0}. A comparison with numerical solutions of system generalizing (2) for a complex kinetics scheme shows that the thus obtained nonlinear laws (9b) are satisfactory for real mixtures such as hydrogen and oxygen [6]. Note that the quantity $\exp[-\Theta_N(\tau-\xi)]$ in (9b) corresponds to the induction time scale of the gases propagating from the shock with the entropy waves.

C. Method

Once the instantaneous distribution of the heat release rate is known in terms of the history of the shocked gas temperature $T_N(\tau)$, as given by (9b) and (9c), the detonation dynamics is obtained as follows. The unknown function is the reduced temperature fluctuation $\Theta_N(\tau)$ which, according to (1), represents the motion of the shock, $M(\tau)$. The acoustic modes are fed by the heat release variations (9b) appearing as a source term in equations (4). The upstream propagating wave $(-)$ is responsible for a feedback upon the leading shock, while the downstream one $(+)$ determines the resulting amplitude of the acoustic fluctuations transmitted to the burned gases. In principle, an integral equation for the evolution of $\Theta_N(\tau)$ is obtained by a space integration of (4) along the wave $(-)$ propagating upstream from the burned gas at $\xi = +\infty$ where condition (6) holds, toward the shock at $\xi = 0$ where (1a) and (5) are satisfied. The variations of the sound speed prevents us from analytically carrying out such an integration in the general case. Particular regimes are worth considering.

III. DIFFERENT REGIMES

The problem may be formulated in another equivalent form, convenient to identify further approximations. Equations (4) result from mass and momentum equations which may be written in terms of reduced variables (3) and reduced functions $r \equiv \rho_{N0}/\rho$, $u \equiv v/v_{N0}$, and $\pi \equiv p/p_{N0}$ as

$$-\frac{D}{D\tau}r + \frac{\partial}{\partial\xi}u = 0, \quad \varepsilon^2\frac{D}{D\tau}u + \frac{\partial}{\partial\xi}\left(\frac{\theta}{r}\right) = 0, \quad (10a)$$

where the equation of ideal gas, $\pi \equiv \theta/r$, has been used, and where, by definition,

$$\varepsilon^2 \equiv \gamma M_{N0}^2. \quad (10b)$$

The local Mach number of the flow at the Neumann spike of the unperturbed solution, $M_{N0} \equiv v_{N0}/a_{N0}$, is given by the Rankine-Hugoniot conditions written in a strong shock approximation (1b) as

$$\varepsilon^2 = \gamma M_{N0}^2 \simeq \frac{2+(\gamma-1)M_0^2}{2M_0^2}. \quad (11a)$$

According to (8a) and (8b), ε^2 is a small number of the same order of magnitude as $1/\beta_N$,

$$\varepsilon^2 = O\left(\frac{1}{\beta_N}\right). \quad (11b)$$

Once $\theta(\tau,\xi)$, as given by (9b), is introduced into (10a), this system forms a closed set of equations for r and u. The unknown function $\Theta_N(\tau)$ is obtained when the boundary conditions at the shock and at the piston are applied to the solution of (10a). At the leading order of asymptotic expansion (8), boundary conditions (1) and (5) at the shock take the following form:

$$r(\xi=0,\tau)=1, \quad u(\xi=0,\tau)=1-\Theta_N(\tau)/\beta_N(\gamma-1), \tag{12a}$$

while the boundary condition at the piston yields

$$u(\xi=+\infty,\tau)=u_{b0}\equiv v_{b0}/v_{N0}, \tag{12b}$$

where v_{b0} is the prescribed piston velocity supporting the detonation in the burned gases at $\xi=+\infty$. Unfortunately the analytical solution of (10a) for a given function $\theta(\tau,\xi)$ is not an easy task in general.

The presence of the small ε^2 term suggests a perturbative approach. However, this term does not always yield a small contribution. This ε^2 term cannot generally be neglected at the leading order of asymptotic expansion (8), because $Du/D\tau$ may take values as large as $1/\varepsilon^2$. The condition $\varepsilon^2 \ll 1$ means that the flow just downstream of the shock is very subsonic. However, the local Mach number of the flow increases with the distance from the shock under the effect of the heat release, and the ε^2 term is negligible in (10a) only in a limiting cases when a quasiisobaric approximation is valid everywhere downstream of the shock.

A. Steady solutions

The steady solutions of (10a),

$$u_0(\xi)=r_0(\xi)=\frac{1}{2\varepsilon^2}\left[(1+\varepsilon^2)-\sqrt{(1+\varepsilon^2)^2-4\varepsilon^2\theta_0(\xi)}\right], \tag{13a}$$

are useful to classify the different regimes of propagation. The temperature increases monotonously with the distance from the shock, $\theta_0(\xi)\in[1,\theta_{b0}]$, as well as the local Mach number, an increasing function of the temperature. Within the framework of approximations (8a) and (9c), the leading order of the burned gas temperature at $\xi=+\infty$ is given by

$$\theta_{b0}\equiv 1+q \quad \text{with} \quad q\equiv Q/C_pT_{N0}. \tag{13b}$$

According to (13a), the reduced heat release is bounded from above by q_{CJ}, $q\leqslant q_{\mathrm{CJ}}$, with, by definition,

$$\theta_{b\mathrm{CJ}}=1+q_{\mathrm{CJ}}\equiv(1+\varepsilon^2)^2/4\varepsilon^2\approx 1/4\varepsilon^2. \tag{14a}$$

The case $q=q_{\mathrm{CJ}}$ corresponds to the marginal Chapman-Jouguet (CJ) regime with, according to (13a),

$$u_{b0}=u_{b\mathrm{CJ}}\equiv(1+\varepsilon^2)/2\varepsilon^2\approx 1/2\varepsilon^2, \tag{14b}$$

yielding a sonic condition at the end of the reaction $v_{b0}=a_{b0}$, with $a_{b0}/a_{N0}=\theta_{b0}^{1/2}$. In practical situations, the detonation regime M_0 is prescribed by the piston velocity v_{b0} for any given initial thermodynamic state (p_u,T_u) of the fresh mixture. The CJ wave corresponds to the minima of v_{b0} and M_0, $v_{b0}\geqslant v_{b\mathrm{CJ}}$, and $M_0\geqslant M_{\mathrm{CJ}}$. There is no steady solution for $v_{b0}\leqslant v_{b\mathrm{CJ}}$, and the Mach number of the detonation M_0 increases with v_{b0}. The CJ minima may be computed in terms of (p_u,T_u) from (14) and the Rankine-Hugoniot conditions to give, at the leading order of (8a), $M_{\mathrm{CJ}}^2=4Q/C_pT_u$.

B. Independent parameters

The overdrive factor, defined as $f\equiv(M_0/M_{\mathrm{CJ}})^2\geqslant 1$, is extensively used in the specialized literature to classify the different regimes of steady propagation. Neither the initial thermodynamic parameters (p_u,T_u) nor the overdrive factor f appear explicitly in our formulations (10) and (12). Two independent parameters (ε^2,q), defined by (11a) and (13b), characterize both the reactive mixture and the steady propagation regimes. The q parameter appears through u_{b0} in (12b), which is, according to (13a), a function of q and ε^2. The correspondence between parameters (ε^2,q) and (f,M_{CJ}^2), is given by the Rankine-Hugoniot conditions

$$f=\left(\frac{\gamma+1}{2}\right)\frac{1}{4q\varepsilon^2}, \quad M_{\mathrm{CJ}}^2=\frac{1}{\left(\varepsilon^2-\dfrac{\gamma-1}{2}\right)f}, \tag{15}$$

showing, as in (11a), that strong shock assumption (1b) leads to small ε^2, $\varepsilon^2=O(\gamma-1)$, but with $\varepsilon^2>(\gamma-1)/2$.

Two additional parameters $[w_0(\xi), \beta_N(\gamma-1)]$ characterizing the chemical kinetics, appear in the analysis. The reduced distribution $w_0(\xi)$ of the heat release rate governs the steady temperature profile $\theta_0(\xi)$ in (10a), while the temperature sensitivity of the induction kinetics, $\beta_N(\gamma-1)$, appears in (12a). These kinetics parameters govern the dynamical properties of detonations for any regime of steady propagation characterized by parameters (ε^2,q).

C. Different regimes of propagation

The different regimes of steady propagation are classified by the relative ordering between q and ε^2 or, by using (11b) in the limit (8a), between q and β_N. Two extreme regimes are worth considering first: near CJ regimes (low overdrive)

$$q=O(\beta_N), \tag{16a}$$

and regimes at high overdrive,

$$q=O(1). \tag{16b}$$

As for the CJ regime (14a) and (14b), the near-CJ regimes defined by (16a) are characterized by $q=O(1/\varepsilon^2)$, yielding, according to (15), $f=O(1)$. According to (13a) and (13b), one has $u_{b0}=O(1/\varepsilon^2)$ and the variations of u, r, and θ across the detonation structure are large, of the same order of magnitude as $1/\varepsilon^2$, while π varies of order unity. The local Mach number of the flow reaches values close to unity. The ε^2 term cannot be neglected in (10a), $\varepsilon^2 Du/D\tau=O(1)$. The opposite regimes defined by (16b) correspond to a large overdrive factor $f=O(1/\varepsilon^2)$. Here the variations of u, r, and θ across the detonation structure are, according to (13a), of order unity, while the variations of π are small of the same order of magnitude as ε^2 [see (10a)]. Thus, at the leading order of limit (8a), the ε^2 term is fully negligible in (10a), and a quasi-isobaric approximation is valid throughout the detonation structure.

D. Detonation dynamics at high overdrives

A nonlocal equation for $\Theta_N(\tau)$ describing the galloping detonations in these regimes has recently been obtained [6]. The main steps are recalled here. Generally speaking, the mass flux across the shock must be balanced by the integral of the instantaneous rate of density change. A space integration of the equation for the mass conservation, first equation (10a), with boundary conditions (12a) and (12b), yields

$$\frac{(v_{b0}-v_{N0})}{v_{N0}}+\frac{\Theta_N(\tau)}{(\gamma-1)\beta_N}=\int_0^\infty \frac{D}{D\tau} r\, d\xi. \qquad (17a)$$

Finally the quasi-isobaric approximation $r\approx\theta$, and (9a) and (9c), $Dr/D\tau\approx D\theta/D\tau\approx qw$, leads to the following nonlinear integral equation:

$$1+b\Theta_N(\tau)=\int_0^\infty e^{\Theta_N(\tau-\xi)}w_0(\xi e^{\Theta_N(\tau-\xi)})d\xi,$$

with (17b)

$$b\equiv 1/q\beta_N(\gamma-1)=O(1),$$

valid at the leading order of an asymptotic expansion (8a) and (8b) for the dynamics of detonation in the regime defined by (16b) and where $q\beta_N(\gamma-1)$ is a parameter of order unity. Oscillatory instabilities with a negligible amplitude of pressure waves are described by (17b) with a period of oscillation related to the transit time of fluid particles from the shock to the end of the reaction and, thus, according to (11a) and (11b), much longer than the acoustic time. This shows that galloping detonations are not related to ordinary thermoacoustic instabilities, at least in limiting case (16b) [6].

The rest of this paper is devoted to studying more realistic regimes at moderate overdrive, in between (15a) and (15b), in which the effect of pressure waves is no longer fully negligible but small enough to be investigated by a perturbation analysis.

IV. ANALYSIS AT MODERATE OVERDRIVES

When approaching the marginal CJ regime from the large overdrive ones by increasing q from order unity to large values [see (16a) and (16b)], the Mach number of the burned gases increases. However, according to (11a) and (11b), the local Mach number of the shocked gases is kept small in the limits (8a) and (8b). If the local Mach number remains small on a sufficiently long distance from the leading shock, the transit time of entropy waves is still longer than the one of the acoustic mode propagating downstream. Then the position of the peak reaction rate fluctuates with a much smaller velocity than the sound speed propagating downstream; strong thermoacoustic instabilities are ruled out. The instability mechanism is expected to be similar to the quasi-isobaric one but with quantitative changes. In particular, the upstream propagation of pressure signals must introduce an additional time delay, further increasing the period of oscillation.

A. Order of magnitude estimates

Orders of magnitude are obtained from the acoustic mode propagating upstream given by (4)

$$\left[(\alpha-m)\frac{\partial}{\partial\xi}-\frac{\partial}{\partial\tau}\right]\left(u-\frac{\theta^{1/2}}{\gamma M_{N0}}\ln\pi\right)=\alpha H, \qquad (18a)$$

with

$$H(\xi,\tau)\equiv\frac{q}{\pi}w(\xi,\tau)-\frac{\ln\pi}{\gamma M_{N0}}\left(\frac{\partial}{\partial\xi}-\frac{1}{\alpha}\frac{D}{D\tau}\right)\theta^{1/2}, \qquad (18b)$$

where $w(\xi,\tau)$ is given by (9b), $m\equiv M(\tau)/M_0$ is the same factor as in (3), and $\alpha=(M_{N0}\theta^{1/2})^{-1}\pi$, introduced in (4), is expressed here for an ideal gas. An integration along the characteristic curve, $\xi=x(\tau)$ with $dx/d\tau=-(\alpha-m)$, yields

$$\left[u-\frac{\theta^{1/2}}{\gamma M_{N0}}\ln\pi\right]_{\xi=0}^{\xi}=\int_0^\xi H(x',\tau-\Delta\tau)\alpha\frac{dx'}{(\alpha-m)}, \qquad (19a)$$

where $\Delta\tau$ is the time delay on this characteristic curve,

$$\Delta\tau\equiv\int_0^{x'}\frac{dx''}{(\alpha-m)}. \qquad (19b)$$

At the marginal CJ case the denominator in (19b) vanishes at the end of the reaction but without divergence of the integral. Thus the order of magnitude of the additional time delay across the detonation thickness is easily obtained from (19b), $\Delta\tau=O(M_{N0}\theta_{b0}^{1/2})$. At high overdrives (16b) this time delay is, according to (8b) and (13b), $\theta_{b0}=O(1)$, of the same order of magnitude as $M_{N0}=O(1-\gamma)^{1/2}$, and $\Delta\tau=O[(1-\gamma)^{1/2}]$, yielding, in the framework of (8a), (11a) and (11b), a correction to (17b) of order $\beta_N^{-1/2}$. However, in the opposite case, for near CJ regimes (16a), $\Delta\tau$ is, according to (11a), (11b), and (14a), $\theta_{b\mathrm{CJ}}\approx 1/4M_{N0}^2$, of order unity $M_{N0}\theta_{b\mathrm{CJ}}^{1/2}\approx 1/2$, yielding a correction to (17b) of order unity.

B. Analysis

Here we study the intermediate regime at moderate overdrive, in between (16a) and (16b), and defined by

$$1\leqslant q<\beta_N, \qquad (20a)$$

which, according to (11a), (11b), and (13b), also corresponds to

$$\beta_N^{-1/2}\leqslant M_{N0}\theta_{b0}^{1/2}<1, \qquad (20b)$$

where these inequalities have to be understood as concerning the orders of magnitude. According to (13a), the variations of u_0 and θ_0 across the detonation structure are of the same order as q, while, as shown below, the variation of π is still small, of order $M_{N0}^2\theta_{b0}$. The acoustic effects are described by working at the first order of an expansion in powers of the small parameter $M_{N0}\theta_{b0}^{1/2}$ in the limits (8a) and (8b) by neglecting term of order β_N^{-1} and still using (9a)–(9c). Regime (20a) ranges from high overdrives (16b) to moderate overdrives. Every propagation regime characterized by an overdrive factor f of the same order of magnitude as $\beta_N^{1/n}$,

$f = 0(\beta_N^{1/n})$, with a positive integer $n \geqslant 1$, may be described by this perturbative analysis, up to the first order, $M_{N0}\theta_{b0}^{1/2} = O(\beta_N^{-1/2n})$, neglecting terms of second order, $M_{N0}^2\theta_{b0} = O(\beta_N^{-1/n})$. This shows that terms of order β_N^{-1} may be consistently neglected, and approximations (9a) and (9c) may still be used. Such an analysis provides a satisfactory description of regimes not too far from the marginal CJ condition, which is approached for large n, $n \gg 1$.

The local Mach number increases with the distance from the shock but, in propagation regime (20a), it keeps a value sufficiently smaller than unity to work in perturbation around the quasi-isobaric approximation of the detonation structure, $M_{b0} = M_{N0}\theta_{b0}^{1/2}$ and $\alpha^{-1} = O(M_{N0}\theta^{1/2})$. The order of magnitude of pressure variation is evaluated from the second equation of (10a) by anticipating that the period of oscillation keeps the same order as at high overdrives, yielding $\delta\pi = O(M_{N0}^2\theta_{b0})$. The second term on the right-hand side of (18b) describes gradient effects of the nonhomogeneous medium across which the sound propagates, yielding a relative correction to the first term qw of order $M_{N0}q/\theta_{b0}^{1/2}$ which is of the same order of magnitude as the first relative correction to the delay, $M_{N0}\theta_{b0}^{1/2}$. The leading order of the pressure variation across the detonation is obtained by introducing the quasi-isobaric approximation of u in the equation for momentum conservation [the second equation of (10a)]

$$\frac{[\pi(\xi,\tau) - \pi(\xi=0,\tau)]}{\gamma M_{N0}^2} = -q\int_0^{\xi} w(\xi',\tau)d\xi' + q\int_0^{\xi} d\xi' \int_{\xi'}^{\infty} \frac{\partial w(\xi'',\tau)}{\partial \tau}\, d\xi'', \tag{21}$$

where $\delta\pi(\xi=0,\tau)$ is given by (1a).

According to (4) with $\alpha^{-1} \approx (M_{N0}\theta^{1/2})$, the ratio of the detonation thickness to the acoustic wavelength in the burned gases is small, of order $M_{N0}\theta_{b0}^{1/2}$. Pressure field (21) is valid across the detonation structure, a thin layer compared to the acoustic wavelength. In the framework of a multiple scale approximation associated with (20b), the limit $\xi \to +\infty$ of (21) yields the value of the acoustic field at the entrance of the burned gas region where a radiation condition (6) holds. As H vanishes outside the detonation thickness, a linear expansion in $\Delta\tau$ may be introduced into (19a) with, according to (19b), is

$$\Delta\tau \approx M_{N0}\int_0^{x'} \theta(x'',\tau)^{1/2} dx''. \tag{22}$$

Then the same quasi-isobaric mass conservation as at high overdrives is obtained by introducing the limit $\xi \to +\infty$ of (21) into (19a),

$$\frac{u(\xi=+\infty,\tau) - u(\xi=0,\tau)}{q} = \int_0^{\infty} w(\xi',\tau)d\xi', \tag{23}$$

but valid here up to the first order, $M_{N0}\theta_{b0}^{1/2}$, and limited to the detonation structure. This is easily understood by noting that the pressure term in the mass conservation [first equation of (10a)] yields a correction to u/q of the following order, $M_{N0}^2\theta_{b0}$, so that $-\partial u/\partial\xi = Dr/D\tau \approx D\theta/D\tau \approx qw$ is valid

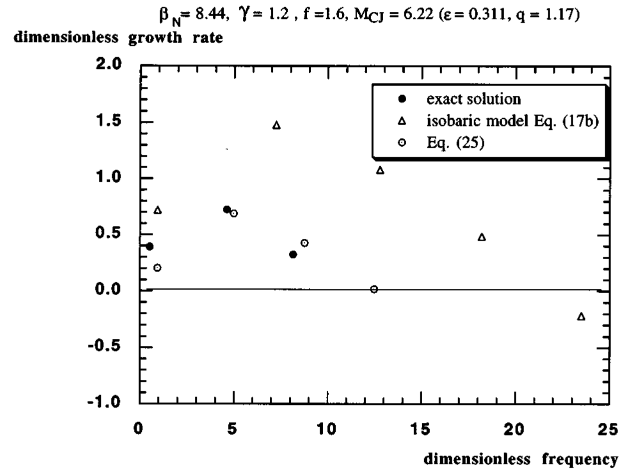

FIG. 2. Dimensionless growth rate vs dimensionless frequency of the eigenvalues of a typical unstable spectrum obtained from Eqs. (25a) and (25b) in comparison with both the exact numerical solutions of the linearized version of Eqs. (2)–(4) with boundary conditions (1a), (5), and (6) and the approximate solution obtained from the quasi-isobaric model (17b). The considered parameters are $\gamma = 1.2$, $f = 1.6$, and $M_{\mathrm{CJ}} = 6.22$ ($\varepsilon = 0.311$ and $q = 1.17$). The reaction model is an Arrhenius law with $\beta_N = 8.44$. The time scale used to reduce the variables is the half-reaction time.

inside the detonation thickness up to the first order. However, the velocity fluctuations at the exit of the detonation structure, $\delta u(\xi = +\infty,\ \tau)$, feed acoustic waves in the downstream region of burned gases with a velocity amplitude $\delta u_b/u_{b0}$ of order $M_{N0}\theta_{b0}^{1/2}$ and a pressure amplitude $\delta\pi$ of order $M_{N0}^2\theta_{b0}$. Then the first pressure correction to the detonation dynamics is obtained from (23) by using as boundary condition $u(\xi = +\infty,\ \tau)$, the acoustic radiation condition (6), written as

$$u(\xi=+\infty,\tau) - u_{b0} = \theta_{b0}^{1/2}[\pi(\xi=+\infty,\tau) - \pi_{b0}]/\gamma M_{N0}, \tag{24}$$

where $\delta\pi(\xi = +\infty,\ \tau)/\gamma M_{N0}$ is given by the limit $\xi \to +\infty$ of (21) in which $\delta\pi(\xi = 0,\ \tau)$ is obtained from Rankine-Hugoniot condition (1a). Another Rankine-Hugoniot condition (5) provides $u(\xi = 0,\ \tau)$ in (23). The final result for the detonation dynamics at intermediate regimes (20a) and (20b) may be written in the form of a nonlinear integral equation, similar to (17b),

$$1 + b'\Theta_N(\tau) = \int_0^{\infty} e^{\Theta_N(\tau - \kappa'\xi)} w_0(\xi e^{\Theta_N(\tau - \kappa'\xi)}) d\xi, \tag{25a}$$

valid up to the first order, $M_{N0}\theta_{b0}^{1/2}$, but with both a modified bifurcation parameter b' and a modified delay $\kappa'\xi$ resulting from pressure effects,

$$b' \equiv b(1 + M_{N0}\theta_{b0}^{1/2}), \quad \kappa' \equiv (1 + M_{N0}\theta_{b0}^{1/2}), \tag{25b}$$

where parameter b is the same as in (17b).

V. DISCUSSION AND CONCLUSION

As shown in [6], Eq. (17b) presents a Hopf bifurcation for a sufficiently strong temperature sensitivity (large β_N, small

b) representing the galloping detonations of strongly overdriven detonations (16b). The reduced distribution $w_0(\xi)$ is stiffer, the critical value of β_N is smaller, and the detonation is more unstable. The origin of the oscillations lies on the delay introduced by the entropy waves in a quasi-isobaric mass conservation. The nonlinear behavior is governed by the temperature sensitivity of the induction kinetics. Comparisons with numerical simulations show right orders of magnitude for both frequencies and critical values of parameter b obtained from (17b), but with quantitative discrepancies at moderate overdrives [6]; see Fig. 2.

According to (25a) and (25b), describing the dynamics of detonations at moderate overdrives defined by (20a) and (20b), the pressure fluctuations introduce two quantitative modifications from the results at high overdrives but no qualitative change: (i) The period of the oscillation is increased, and (ii) the critical value of the bifurcation parameter b is decreased. By increasing the critical value of the temperature sensitivity β_N at the instability threshold [the critical value of $1/q\beta_N(\gamma-1)$ is decreased], the pressure effects appear to be stabilizing. The main reason is that the energy rate involved by the pressure fluctuations at the shock is, according to (1a) and (5), negative, $\delta p_N \delta v_N < 0$, exhibiting a damping mechanism which is added to the one associated with the radiation condition at the end of reaction (6). These two damping mechanisms overcome the other pressure effects, so that the net effect of the pressure is stabilizing. The destabilizing mechanism at moderate overdrives is still the quasi-isobaric mass conservation exhibited at high overdrives. This result definitely clarifies the nature of galloping detonations at ordinary conditions.

The oscillatory instability of detonations very close to the CJ regimes represented by (16a) cannot, from a mathematical point of view, be described by the perturbative analysis used for regimes represented by (20a). The reason is that the perturbation parameter $M_{N0}\theta_{b0}^{1/2}$ has an order of magnitude which is no longer smaller than unity, $M_{N\mathrm{CJ}}\theta_{b\mathrm{CJ}}^{1/2}=\frac{1}{2}$. However, as no new phenomenon is involved, only quantitative modifications are expected. Moreover, the neglected terms being typically $M_{N\mathrm{CJ}}^2\theta_{b\mathrm{CJ}}=\frac{1}{4}$, the quantitative differences are not so large. Comparisons with numerical results show a relatively good quantitative agreement for both the period of oscillations and the stability limits predicted by (25) at moderate overdrives. The results are still satisfactory for overdrive factors which are not very far from unity. As an example, a typical unstable spectrum given by (25) is plotted in Fig. 2, in comparison with both the exact numerical solutions and the approximation solutions obtained from the quasi-isobaric model.

Owing to the effects of inhomogeneities, the additional delay (25b), $M_{N0}\theta_{b0}^{1/2}$, is quantitatively different from (19b) but has the same order of magnitude. The same equation as (25a) involving the same delay as (19b) is obtained when the gradient effects are negligible, as in the limiting case of a small heat release, $q<1$, corresponding to very strongly overdriven detonations [8].

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